1.10 Mathematical Modeling and Variation

Direct Variation:

\[ y = kx \] implies that \( y \) varies directly as \( x \) or that \( y \) is directly proportional to \( x \). \( k \) is the constant of variation or the constant of proportionality.

The simple interest on an investment is directly proportional to the amount of the investment. By investing $2500 in a certain bond issue, you obtained an interest of $187.50 at the end of 1 year. Find the mathematical model that gives the interest \( I \) for this bond issue at the end of 1 year in terms of the amount invested \( P \).

Direct Variation as an nth Power:

\[ y = kx^n \] implies that \( y \) varies directly as the \( n \)th power of \( x \) or \( y \) is directly proportional to the \( n \)th power of \( x \).

Neglecting air resistance, the distance \( s \) an object falls varies directly as the square of the duration of the fall. An object falls a distance of 144 feet in 3 seconds. Write an equation relating the distance \( s \) and duration \( t \).

How far will an object fall in 6 seconds?
Inverse Variation:

\[ y = \frac{k}{x} \text{ implies that } y \text{ varies inversely as } x \text{ or that } y \text{ is inversely proportional to } x. \]

A company has found that the demand for its product varies inversely as the price of the product. When the price is $2.75, the demand is 600 units. Write an equation relating demand \( d \) and price \( p \).

Approximate the demand when the price is $3.25.

Joint Variation:

\[ z = kxy \text{ implies that } z \text{ varies jointly as } x \text{ and } y \text{ or that } z \text{ is jointly proportional to } x \text{ and } y. \]

The maximum load that can be safely supported by a horizontal beam is jointly proportional to the width of the beam and the square of its depth, and inversely proportional to the length of the beam. Determine the change in the maximum safe load when the width is doubled.

How about when the depth of the beam is doubled?
Write a mathematical model:

- \( r \) varies directly as the square of \( u \):

- \( h \) varies directly as \( N \) and inversely as \( p^2 \):

- \( Q \) is jointly proportional to the cube of \( t \) and the square root of \( L \):

Find \( k \) if \( Q = 270 \) when \( L = 4 \) and \( t = 3 \)

What is the final model?

\[ W \] is jointly proportional to \( b \) and the difference between \( b \) and \( Y \).

Find \( k \) if \( W = 24 \) when \( b = 4 \) and \( Y = 6 \).

The circumference of a circle is given by \( C = \pi d \). Write a sentence using variation terminology to describe the formula.